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CONTROL THEORY AND PARTIAL DIFFERENTIAL EQUATIONS

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PROGRESS REPORT (Dec., 1983)

This describes research activity supported under this grant from its inception (June, 1982) to the present. (References are to the attached bibliography.)

PART I: Published and other completed papers

During this period 8 papers by the P.I. were published (1982b,c,d,e,f; 1983a,b,c); one additional paper (1983d) was accepted earlier and is expected to appear in 1983.

Research supported by the grant appears in two of these (1982e; 1983c) and in an additional 9 papers (1982h; 1983e,f,g,h,i,j,k,l) which have been written but not yet published; all but the last of these have already been typed. Brief descriptions follow.

> 1982e: The interior state of a one-phase Stefan problem (no input, unknown initial state) depends continuously on observations of the moving boundary so state identification is well-posed.

> 1982h; 1983c,g: Some new harmonic analysis results (1982h) giving estimates for the coefficient map:

$$f(t) = \sum c_k^{\pm} \exp(\pm i(k^2+\nu)t) \mapsto (c_k^{\pm})$$

from $L(-\delta,\delta) \to \ell^2$ can be used (1983c,g) to obtain results on boundary observability and controllability in arbitrarily short time for a rectangular vibrating plate given by $\ddot{u} + \Delta^2 u = 0$. (The paper (1982h) has been submitted to SIAM J. Math. Anal., (1983c) appeared in the Proceedings of the 1982 Vorau Workshop on DPS, and (1983g) has been submitted to Appl. Math. & Optim.)

> 1983f: This revision of an earlier version contains new results on average power boundedness, ($\Sigma_1^n |a_j|^r / N < M^r$) for nonautonomous recursively defined input-output maps. The original motivation was an application to adaptive control. A partial extension to continuous time is given which may possibly be relevant, if further extended, to the stability of systems with random intermittent failure of the stabilizing feedback. (Submitted to SIAM J. Math. Anal.)

> 1983h: This paper continues (1980b) in considering a model for diffusion of holes and electrons in a semiconductor device. Treated here are estimates and well-posedness for the mixed IC-BC problem. (Submitted to Nonlinear Anal. -TMA.)

> 1983i: A model is presented for a variable-structure system with quasi-deterministic switching rules (e.g., a thermostat). The primary

concerns are: continuity of dependence on data, stability, existence of periodic solutions. (Submitted to SIAM J. Control/Optim.)

> 1983j: Generalizes (1975c) in permitting spatial variation (hence additional nonlinearity) for $Au = -\nabla \cdot a(\cdot, u, |\nabla u|)\nabla u$. Principal results are on the coercivity of A and on the existence of periodic solutions for $\dot{u}+Au = f(\cdot, u, \nabla u)$ with Neumann data. This work is related to the determination of eddy currents induced in a nonlinearly magnetic material by a periodic external field. (Submitted to J. Diff. Eqns.)

> 1983k: Explores a class of functions introduced by Ptak for analyzing nonlinear recursions.

> 1983l: A new derivation of a stabilizing feedback law for a one-dimensional parabolic equation, introducing a semigroup involving intervals of memory for observations and controls. The general construction should also be applicable to other situations. (This was presented at the Norfolk SIAM meeting and has been written up and submitted for the biennial INRIA Symposium on Optimization and Systems -- June 19-22, 1984.)

PART II: Additional results

The following represent anticipated papers for which the results have already been obtained although the actual writing has not yet been done. They have thus been included in the bibliography with 1984 numbers. (In some cases parenthetical indication is given of possible continuation.)

> 1984a: Combines a more detailed exposition of material (on the design of stabilizing boundary control by feedback using interior point observations and boundary observation) from (1982f; 1983l) with some new related results.

> 1984b: Continues the line of research of (1980b; 1983h) to discuss boundedness of solutions with bounded data and existence of periodic solutions for a standard model from semiconductor device theory.

(As a longer term project, it is hoped to be able, eventually, to include in the model such effects as avalanche breakdown (IMPATT devices) and negative resistance, as well as to return to consideration of numerical methods for such problems.)

> 1984c: Considers continuous dependence of the physically significant $H=a(\cdot, |\nabla u|)\nabla u$ for a degenerate version of (1983j) arising in a computational approximation proposed for the penetration problem by Bossavit/Damlamian.

(For the future: The treatment in (1975c; 1983j; 1984c) corresponds to physics only for special geometries where the vector potential has a single non=zero component; one would want to extend these results to suitable systems. This is under discussion with I. Mayergoyz. Also, a numerical method, related to (1973b), will be explored.)

> 1984d: (with T. Suzuki) A new convergent algorithm, related to (1981b,c), for recovering the coefficient in a Sturm-Liouville problem from spectral data.

(It is hoped to continue by trying to extend the Gel'fand-Levitan theory for this problem to certain higher dimensional problems which reduce to one dimension by symmetry -- e.g., radial problems for a sphere. Some work on this is now being done in Tokyo.)

> 1984e: An improvement on the results of (1982e); results for the two-phase problem will be included if obtained.

> 1984f: An abstract 'projection' result related to LQ-optimal control with one-sided constraints -- as, e.g., positivity of the control.

> 1984g: Unique identifiability from boundary observations of the nonlinearity $D(\cdot)$ in $\dot{u} = \nabla \cdot D(u)\nabla u$. (It is hoped to extend these results jointly with C. Dafermos.)

PART III: Some other work in progress

> (with W. Krabs) We hope to continue work on boundary control of a vibrating plate. In particular, we hope to consider more general geometries and to meet before or during the Vorau workshop to work out details. Somewhat more difficult is the approach to the genuine nonlinear problem (for small displacements).

> (with A. Becker) On stochastic convergence of an identification algorithm in linear, discrete-time adaptive control. (Convergence has been shown for a related simpler algorithm by Becker/Kumar/Wei.)

> (with N. U. Ahmed) On patch observation and control for a diffusion equation -- cf, (1977b).

> (with S. Belbas) On periodic solutions of a quasi-variational inequality related to optimal stochastic control.

> It is (still) hoped that certain ideas can be explored for extension of the notion of 'viscosity solution' to certain Hamilton-Jacobi-Bellman equations in Hilbert space related to stochastic control or to variable-structure systems (control by 'switching' modes) for parabolic equations. (Other approaches have been introduced by Barbu and da Prato; it is hoped to discuss these further with them, perhaps at Nice or Vorau.)

> Development of effective numerical methods for parabolic equations with piecewise smooth data -- e.g., failing to satisfy consistency conditions at the 'parabolic corner': $t=0$, $x \in \partial\Omega$, as occurs for LQ-optimal boundary control of the heat equation, or with discontinuities as in bang-bang control for time-optimality.

> Etc.

b. Convergent approximation methods for ill-posed problems, Part I--General theory, Control and Cybernetics 10 (1981) pp. 31-49.

c. Convergent approximation methods for ill-posed problems, Part II-Applications, Control and Cybernetics 10 (1981) pp. 51-71.

d. (with P. R. Kumar) On the optimal solution of the one-armed bandit adaptive control problem, IEEE Trans AC-26 (1981) pp. 1176-1184.

1982 a. Boundary inequalities for eigenfunctions and boundary control theory, J. Diff. Eqns. 44 (1982) pp. 452-459.

b. Approximation methods for distributed parameter systems, Chapter 9 of Distributed Parameter Control Systems (edited by S. G. Tzafestas), Pergamon, N.Y., 1982, pp. 273-

c. Regularity of optimal boundary controls for parabolic equations, I: analyticity, SIAM J. Control & Opt. 20 (1982) pp. 428-453.

d. (with H.-X. Zhou) Existence and uniqueness of optimal controls for a quasilinear parabolic equation, SIAM J. Control & Opt. 20 (1982) pp. 747-762.

e. State estimation for a Stefan problem, Proc. 21-st IEEE Conf. on Decision & Control, IEEE, N.Y., 1982, pp. 1082-108

f. Stabilization of nonlinear parabolic equations, Proc. 3-rd IFAC Symposium on Control of Distributed Parameter Syst. (Toulouse, 1982).

g. (with S. Sethi, N. Derzko) Dynamics and optimization of a sales-advertising model with population migration, UMBC report MRR-82-1.

h. The coefficient map for certain exponential sums, UMBC report MRR-82-21.

1983 a. Existence of optimal controls for some nonlinear distributed parameter problems, Proc. 1983 Conf. on Inf. Sci. and Syst., Johns Hopkins Univ., Baltimore, 1983, pp. 588-590.

b. Existence and regularity of extrema, J. Math. Anal. & Appl. 94 (1983), pp. 470-478.

c. Boundary observation and control of a vibrating plate: a preliminary report, in Control Theory for Distributed Parameter Systems and Applications. (Lecture Notes in Control & Inf. Sci., Vol. 54) (Proc. of 1982 Vorau Workshop; edited by F. Kappel, K. Kunisch, W. Schappacher), Springer-Verlag, Berlin, 1983, pp. 208-220.

1983 (cont'd)

d. Two results on exact boundary control of parabolic equations, Appl. Math & Opt. (1983) to appear.

e. L^{∞} bounds for solutions of parametrized elliptic equations, J. Diff. Eqns., to appear.

f. (with P. R. Kumar) Stability in the sense of bounded average power, UMBC report MRR-83-5.

g. (with W. Krabs) On Boundary controllability of a vibrating plate, Appl. Math & Opt., submitted.

h. (with G. M. Troianiello) Time-dependent solutions of a nonlinear system arising in semiconductor theory, UMBC report MRR-83-6.

i. Switching systems: thermostats and periodicity, UMBC report MRR-83-7.

j. A class of nonlinear elliptic and parabolic problems, UMBC report MRR-83-11.

k. On certain iterative sequences, UMBC report MRR-83-12.

l. Boundary feedback stabilization of a parabolic equation.

Work in progress (results already obtained)

1984 a. Stabilization by boundary control for a one-dimensional parabolic equation.

b. Time-dependent solutions of a nonlinear system arising in semiconductor theory, II: boundedness and periodicity.

c. A continuous dependence result for a degenerate periodic parabolic equation.

d. (with T. Suzuki) Convergence of an approximation algorithm for the inverse Sturm-Liouville problem.

e. Observation and prediction for a one-phase Stefan problem.

f. A 'nearest point' property with an application to semi-constrained boundary control.